

Algorithmic Efficiency

Big-O Notation



What it is ?

Simplified Analysis of an Algorithm's Efficiency

1. Complexity in terms of input size, N
2. Machine Configuration
3. Logics used in algorithm
4. Time and Space (Memory)

Types of Measurement.....

Worst - Case

Average - Case

Best - Case

General Rules

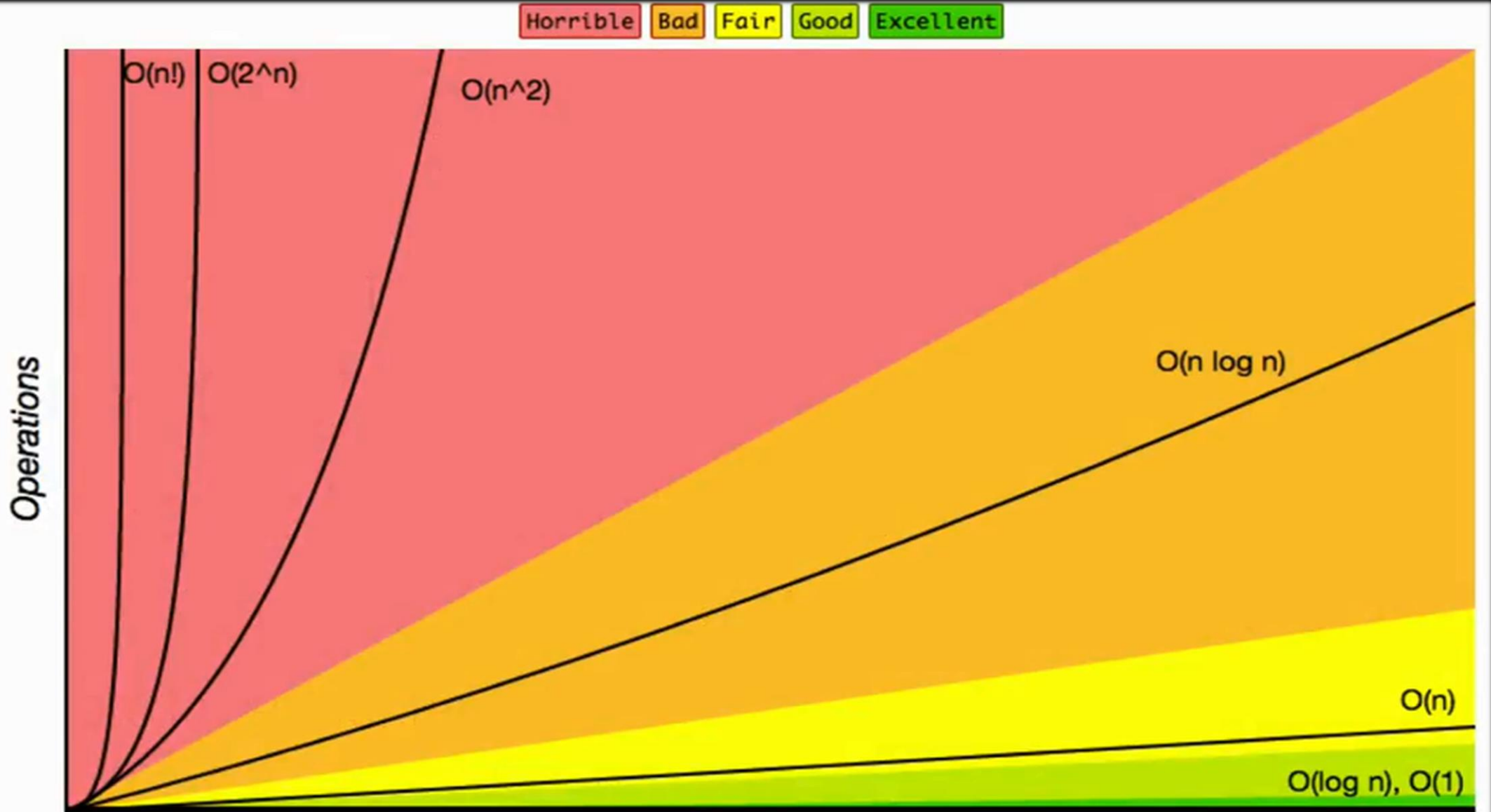
1. Ignore Constant

$$2(n) \rightarrow O(n)$$

2. Ignore Non- dominant terms

$$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n) < O(n!)$$

Big-O Notation Complexity Chart



Constant Time: Complexity

$$X = 2 + (10 * 25)$$

Independent of Input Size (N).

Constant Values may be ignore and
Hence Final time taken will be $O(1)$

Constant Time: Complexity

$$X = 2 + (10 * 25) \quad \Rightarrow O(1)$$

$$Y = 15 - 3 \quad \Rightarrow O(1)$$

$$\text{Print } (X + Y) \quad \Rightarrow O(1)$$

Total Time taken:

$$O(1) + O(1) + O(1) = 3 * O(1)$$

Determine the Bigger (Dominant) term that is $O(1)$

$$\text{Result} = O(1)$$

Linear time: Complexity of loop

for k in range (0,n): \longrightarrow Loop executes "N" times
print (k) \longrightarrow $O(1)$

Total Time taken:

$$N * O(1) = O(N)$$

N is bigger (dominant) term than $O(1)$

So,

$$\text{Result} = O(N)$$

Other Example of Loop

```
y = 5 + (15 * 20);           O(1)
for x in range (0, n):      } O(N)
    print x;
```

total time = $O(1) + O(N) = O(N)$

Total Time taken:

$O(1) + O(N) = O(N)$

for loop term $O(N)$ always dominant other terms

So,

Result = $O(N)$

Quadratic time: Nested loop

```
for X in range (n):      Outer Loop Runs - N Times  
    for Y in range(n): Inner Loop Runs - N Times  
        |  
        print(X * Y ) O(1) times
```

Total Time taken:

$$N * N * O(1) = O(N^2)$$

Nested loop N^2 is bigger (dominant) than $O(1)$

So,

$$\text{Result} = O(N^2)$$

$O(N^2)$

```
x = 5 + (15 * 20);           O(1)
for x in range (0, n):      } O(N)
    print x;
for x in range (0, n):      } O(N2)
    for y in range (0, n):
        print x * y;
```

Total Time taken:

$$O(N^2) + O(N) + O(1) = O(N^2)$$

Nested loop $O(N^2)$ is bigger (dominant) than $O(N)$

and $O(1)$ So,

$$\text{Result} = O(N^2)$$

Calculate Complexity of if-else

To compute the complexity of if-else statement, we consider the worst case running time. Which mean we consider the total time as given below

Time taken by test + time taken by either of block or else part. (Whichever is larger)

Example:

if (A>B):

return False

takes time as constant C₀

takes time as constant C₁

else:

for l in range(n):

if(A<B):

return False

for loop runs n times

takes time as constant C₂

takes time as constant C₃

Time taken
by else part =
(C₂ + C₃) * n

Total time Taken: C₀ + C₁ + (C₂ + C₃) * n that is O(N)

loop N is bigger (dominant) term than other terms in this example.
So, Result will be: **O(N)** (ignore the Constants)